**ATAR Mathematics Methods Units 3 & 4 Exam Notes for WA Year 12 Students**

Created by Anthony Bochrinis Version 1.3 (22/09/17)

**Year 12 ATAR Mathematics Methods Units 3 & 4 Exam Notes**

Exam Notes License Information These exam notes are an attribution, non-commercial, no-derivative work protected by Creative Commons 3.0 Australia. This means that you are free to share, copy and redistribute these exam notes in any medium or format under the following three conditions:

Attribution You must give appropriate credit and provide a link to our website (www.sharpened.com.au) whenever you refer to these exam notes.

Non-Commercial You are strictly not allowed to use these exam notes for any commercial purposes under any circumstances.

No Derivatives If you change, edit or alter the original state of these exam notes, you are not allowed to distribute or shared the modified material under any circumstances.

About the Creator – Anthony Bochrinis I graduated from high school in 2012, completed a Bachelor of Actuarial Science in 2015 and am currently completing my Graduate Diploma in Secondary Education with the goal of becoming a full-time high school teacher next year!

My original exam notes (created in 2013) were inspired by Severus Snape’s copy of Advanced Potion Making in Harry Potter and the Half-Blood Prince; a textbook filled with annotations containing all of the pro tips and secrets to help gain a clearer understanding.

I hope that my exam notes help to sharpen your knowledge and I wish you all the best in your exams!

Using these Exam Notes These exam notes are designed to be a complement to your studies throughout the year. As such, I recommend using these exam notes during class, during tests, whilst studying at home or in the library and even in the calculator-assumed section of your mock and WACE exams.

These exam notes contain theory, diagrams, formulae and worked examples based off the official SCSA syllabus to give you a full revision of the entire course in just 4 pages. For more detailed information about our most frequently asked questions about the use of these exam notes, please visit my website or email me.

Website: www.sharpened.com.au E-Mail Address: support@sharpened.com.au

**... without further ado, I present my exam notes!**

**INDICES / SURDS / LOGARITHMS / EULER’S NUMBER**

**INDEX LAWS**

**SURD LAWS**

LOGARITHM LAWS am×an = am+n am ÷ an = am−n

√a×√b = √ab √a×√a = a

ax = y → x = log

a

a0 = 1 a

√a ÷ √b = √

(y)

ex = y → x = log

e

log

a

(xy) = log

a

(x) + log

a

(y)

log

a

(

x y

) = log

a

(x) − log

a

(y)

**EULER’S NUMBER**

e = 2.718281828...

m n

= (√a n )

m

a

m√a ± n√a = b

(m ± n)√a (am)n = am×n (ab)m = ambm

**RATIONALISING A SURD**

x = log

e

(y)

(y) → x = ln(y)

**EXPONENTIAL LAWS**

log

a

(x)n = nlog

a

**(x) CHANGING LOG BASE (B TO A)**

lim h→0

(

ah h

− 1

) = ln(a)

(

a b

)

m

=

am bm

a−m =

1

√a 1

=

√a 1

×1 =

√a 1

×

√a √a

=

√a a

log

a

(1) = 0 → a0 = 1

log

(x) =

log

b

(x)

lim

(

eh − 1

) =1 R

am

log

a

(a) = 1 → a1 = a

a

log

b

(a)

h→0

h

**S**

**APPLICATIONS OF CALCULUS**

**EXPRESSING DERIVATIVES**

**Type 1st Derivative 2nd Derivative**

y = ⋯

dy dx

= ⋯

d2y dx2

= ⋯

f(x) = ⋯ f′(x) = ⋯ f′′(x) = ⋯

**ATAR Math Methods Units 3 & 4 Page 1 / 4 Created by Anthony Bochrinis - Sharpened®**

**G**

**R**

**A**

**P**

**H**

**I**

**N**

**O**

**G**

**A**

**R**

**I**

**T**

**H**

**E**

**R**

**I**

**V**

**A**

**T**

**I**

**V**

**K**

**E**

**T**

**C**

**H**

**I**

**N**

**C**

**L**

**A**

**S**

**S**

**P**

**A**

**S**

**I**

**N**

**C**

**R**

**E**

**M**

**E**

**N**

**T**

**A**

**D**

**E**

**R**

**I**

**V**

**A**

**T**

**I**

**V**

**U**

**L**

**E**

**S**

**&**

**L**

**A**

**W**

**G**

**L**

**O**

**G**

**A**

**R**

**I**

**T**

**H**

**M**

**S**

**M**

**Q**

**U**

**E**

**S**

**T**

**I**

**O**

**N**

**E**

**R**

**U**

**L**

**E**

**S**

**&**

**L**

**A**

**W**

**S**

**G**

**D**

**E**

**R**

**I**

**V**

**A**

**T**

**I**

**V**

**E**

**S**

**D**

**D**

**E**

**R**

**I**

**V**

**A**

**T**

**I**

**V**

**E**

**L**

**F**

**O**

**R**

**M**

**U**

**L**

**A**

**E**

**Q**

**U**

**E**

**S**

**T**

**I**

**O**

**N**

**S**

**S**

**LOGARITHM FORMULA**

y = log

a

( x − b ) + c

For logarithm curves, there is a condition that a > 1 whereas b and c can take on any value.

x-intercept @ (−1.5,0) Vertical Asymptote @ x = −2

**Solve the equation for x: log**

2

Let x = log

5

**(2) and**

**(b) log**

5

(60) (x) + log

2

(x − 2) = 3

y = log

5

**(3). In terms**

= log

5

(5×22×3) log

2

**of x and y, find:**

= log

5 (a) log

5

Let A = 50log

10

(

35 B

**). How**

(x2 − 2x) = 3 x2 − 2x = 23 = 8

+ log

5 x2 − 2x − 8 = 0

(5) + 2log

5

(2)

**many times greater is B if A is 150 than if A is 250? When A is 150:**

(x − 4)(x + 2) = 0

Solve: 150 = 50log

10 x = 4 or − 2

(18)

(3) = 1 + 2x + y = log

5

(2×32) = log

5

(2) + 2log

5

(3) = x + 2y

**(c) log**

8

(3) =

log log

5

5

y

B = 35,000

(

35 B

) (3) (8)

=

log

log

5

5

(23) (3)

=

3x

**NATURE OF TURNING POINTS**

Type f′(x) f′′(x)

Minimum (Convex) 0 +

Maximum (Concave) 0 −

Horizontal Inflection Point 0 0

Vertical Inflection Point + − 0

**FIND POINT WITH A GIVEN GRADIENT diff(f(x))|x = x co − ord Where:**

• diff is found in Main App → Action → Calculation

• | is found in Keyboard → Math3

• x co − ord is the x co-ordinate of the point given.

**SKETCHING DERIVATIVES**

• All local maximum and minimums are x-intercepts on the derivative.

• Where the function is increasing, the derivative is above the x-axis.

• Where the function is decreasing, the derivative is below the x-axis.

• Where there is a point of inflection on the graph (vertical or horizontal), the derivative has a maximum or minimum turning point.

**FIND GRADIENT OF A POINT solve(diff(f(x))) = gradient f(ans) Where:**

• solve is found in Main App → Action → Advanced

• 1st answer gives the x co-ord and 2nd answer gives y co-ord.

**Find the approximate change in y when x changes from 3 to 2. 98 in the equation: y = 3x2 − 2x δy δx δy ≈ ≈**

dy dx dy dx

×δx

y = 3x2 − 2x dy dx

= 6x − 2

δy ≈ (6x − 2)×δx δy ≈ (6(3) − 2)×(−0.02) δy ≈ −0.32 ∴ decrease by 0.32

**Determine the equation of the tangent to f(x) = ln(5) + x2e at the point f(x) = e.**

Step 1: Find f′(x) f′(x) = 2ex

Step 2: sub x = e into f’(x) f’(e) = 2e(e) = 2e2 2e2 = m in y = mc + c

Step 3: Determine the y co-ord when x = e f(e) = ln(5) + e3

Log graphs have a vertical asymptote at x = b.

When b = 0 and c = 0, the x-intercept of the curve is at (1,0).

**The radius of a sphere increases from 15cm to 15. 1cm, what is the approx increase in surface area? δS δr δS ≈ ≈**

dS

dS dr

dr

×δr

S = 4πr2 dS dr

= 8πr

δS ≈ (8πr)×δr δS ≈ (8π(15))×(0.01) δS ≈ 3.77 ∴ increase by 3.77cm2

( a → 1 Makes curve steeper

**Given y = log**

a

x − 3 )

a → ∞ Makes curve shallower b > 0 Shift curve right by b b < 0 Shift curve left by b

**and (84,4) is a point on the line, find the value of a. y = log**

a

c > 0 Shift curve up by c c < 0 Shift curve down by c

( x − 3 ) 4 = log

a

( 84 − 3 ) 4 = log

a

( 81 ) a = 3

**If Y = log(X), find the ratio of A:B if A is Y when X is 84000 and B is Y when X is 21000000.**

A:B =

7.3222 4.9242 = 1.4870 ∴ A:B = 1:1.4870

**INFLECTION TYPES**

• A vertical point of inflection has a gradient (1st derivative) not equal to 0 and has a 2nd derivative equal to 0. Shown on the right:

• A horizontal point of inflection has a gradient (1st derivative) of 0 and also has a 2nd derivative equal to 0. Shown on the right:

B A

=

=

log(21000000) log(84000)

**DIFFERENTIATION RULES**

**RATES OF CHANGE Type Equation 1st Derivative**

• Instantaneous rate of

Product Rule

y = uv

dy dx

= u′v + uv′

change @ time = t: f'(t)

• Average rate of change

Quotient

between a and b: Rule

u v

u′v − uv′

f(b)−f(a) b−a v2

• Net change in f(x) Chain

between a and b: Rule

∫ |f′(x)|dx y =

dy dx

=

y = [f(x)]n

dy dx

= n[f(x)]n−1×f′(x)

**D**

b a

**FINANCE FORMULAE**

• Profit Formula: P(x) = R(x) − C(x)

• Revenue Formula: R(x) = p(x)q(x)

• Average Cost:

C(x) x

y is positive for all x that is greater than −1. The TP @ x = 1 is a max as the 2nd derivative is negative and the TP @ x = 2 is a min as the 2nd derivative is positive.

Determine the co-ordinates of the second maximum point of f(x) = 2sin(3x) for 0 ≤ x ≤ π f′(x) = 6cos (3x). Solving for x = 0: x = 0.5236,1.5708,2.6180 f′′(x) = −18sin (3x), using 2nd derivative test, 2nd max @ (2.6180,2)

Step 4: Solve for c in y = mx + c, substituting in values for y, m and x

y = mx + c ln(5) + e3 = 2e2(e) + c c = ln(5) − e3

**SECOND DERIVATIVE AT A POINT diff(diff(f(x))) |x = x co − ord Where:**

• Answer gives the value of the second derivative at the point.

f(x)

f′(x) S

Step 5: present the equation of the tangent

y = mx + c m = 2e2 c = ln(5) − e3 y = 2xe2 − e3 + ln (5)

**Find the approximate change in y when x changes from 1 to 1. 1 in the equation: y = sin(2x) + e3x δy δx**

≈

dy dx

y = sin(2x) + e3x

δy ≈

dy dx

×δx

dy dx

= 2cos(2x) + 3e3x

δy ≈ (2cos(2x) + 3e3x)×δx δy ≈ 2cos(2(1)) + 3e3(1)×(0.1) δy ≈ 5.94 ∴ increase by 5.94

**EQUATION OF THE TANGENT tanLine(f(x),x,x co − ord)**

Where:

• tanLine is found in Main App → Calculation → line

• Answer is the linear equation of the tangent in the form of y = mx + c

( y = log

2

**x − a ) + b is below. Find a and b.**

When A is 250: Solve: 250 = 50log

10

)

B = 3,500,000 Determining the Factor:

=

B B

A=250

A=150

3500000 L

=

35000

= 100

**Sketch f(x)**

**a graph and its derivative with the following features:**

• y ≥ 0 for x ≥ −1

• dy/dx = 0 for x = 1, 2

f′(x)

• d2y/dx2 > 0 for x = 2

• d2y/dx2 < 0 for x = 1

For f(x) =

cos(x) 2x+2

**, find f′(π).**

f′(x) =

(− sin(x))(2x+2)−(cos(x))(2) (2x+2)2 f′(x) =

2(−sin(x))(x+1)−2cos (2x+2)2

(x)

f′(π) =

2(π+1)2 1

**CO-ORDS OF A TURNING POINT solve(diff(f(x))) = 0 f(ans) Where:**

• solve is found in Main App → Action → Advanced

• 1st answer gives the x co-ord and 2nd answer gives y co-ord.

The vertical asymptote @ x = −2 means that a = −2 ∴ y = log

2

( x + 2 ) + b

0 = log

2

(−1.5 + 2 ) + b −b = log

2

(0.5 ) ⟹ b = 1

∴ y = log

2

( x + 2 ) + 1

(

B 35

**The radius of a sphere increases by 2%, find the % increase in the volume.**

V dV

=

4 3

πr3

dr

δV ≈ 4πr2×δr

= 4πr2

δV V

≈

4πr2δr V

δV

δV V

≈ 3×

δr r

δV δr

≈

dV dr

δV V

≈

4πr2δr 4πr3/3 δV

3δr

V

≈ 3×2%

δV ≈

dV dr

V

≈

r

δV V

≈ 3×6%

×δr

∴ increase by 6%

**COMMON DERIVATIVES**

**Equation 1st Derivative**

y = axn

dy dx

= n×axn−1

y = ef(x)

dy dx

= f′(x)×ef(x)

y =

1 x

= x−1

dy dx

−1 x2

= −x−2

y = ±sin(x)

=

dy dx

= ±cos(x)

y = ±cos(x)

dy dx

= ∓sin(x)

y = ln[f(x)]

dy dx

f′(x) f(x)

y = ax

=

dy dx

= ln (a)×ax

**APPLICATIONS OF CALCULUS**

**F**

**ATAR Math Methods Units 3 & 4 Page 2 / 4 Created by Anthony Bochrinis - Sharpened®**

**U**

**N**

**D**

**A**

**M**

**E**

**N**

**T**

**A**

**G**

**R**

**O**

**W**

**T**

**N**

**I**

**N**

**T**

**E**

**G**

**R**

**A**

**T**

**I**

**O**

**S**

**D**

**E**

**F**

**I**

**N**

**I**

**T**

**E**

**A**

**N**

**Y**

**O**

**P**

**T**

**I**

**M**

**I**

**S**

**A**

**T**

**I**

**O**

**S**

**R**

**E**

**C**

**T**

**I**

**L**

**I**

**N**

**E**

**A**

**R**

**M**

**O**

**T**

**I**

**O**

**N**

**L**

**T**

**H**

**E**

**O**

**R**

**E**

**M**

**O**

**F**

**C**

**A**

**L**

**C**

**U**

**L**

**U**

**S**

**N**

**R**

**U**

**L**

**E**

**S**

**&**

**L**

**A**

**W**

**D**

**I**

**N**

**D**

**E**

**F**

**I**

**N**

**I**

**T**

**E**

**I**

**N**

**T**

**E**

**G**

**R**

**A**

**L**

**H**

**A**

**N**

**D**

**D**

**E**

**C**

**A**

**GROWTH & DECAY**

• A = A

0

**What is the half-life of the ekt**

**following equation that**

• Where:

dA dt

= kA

0

ekt = kA

**tracks radioactivity of a substance: A = 800e−0.04t?**

• A

0

A = 800e−0.04t 400 = 800e−0.04t t = 17.33 days

OPTIMISATION STEPS Step 1: Draw a diagram of the scenario and define all variables. Step 2: Only 2 variables can be used to optimise a problem, if there are more than 2 variables, reduce the number of variables by substitution and simplification. Step 3: Determine the derivative: diff(f(x)) Step 4: Determine the x co-ord(s) of the turning point(s): solve(diff(f(x))) = 0 Step 5: Determine the nature of the turning point(s): diff(diff(f(x)))|x = x co − ord

**(a) If dA/dt = 0. 252A is an exponential equation, find the initial value for A given that**

→ Initial (starting) amount

• k → continuous rate of change

• A → amount @ time t

A @ time = 10 is 565. A = A

0

ekt → 565 = A

0

e0.252(10) 565/A

0

HALF LIFE Time taken for amount to reduce by 50% (i.e. A = 0.5A

0

= e3.82 = 45.60

**RELATIONSHIP BETWEEN TYPES OF MOTION**

Where:

• a → acceleration

• v → velocity

• s → displacement

).

= A

0

e0.252(10) ln(565/A

0

) = 0.252(10) ln(565) − ln(A

0

) = 2.52 ln(A

0

) = ln(565) − 2.52 = 3.82 A

0

Note: repeat step 5 for each x co-ord found in step 4.

• If

2y

• If

d dx2 2y

> 0 → TP is a min

• If

d dx2 2y

< 0 → TP is a max

of d

dx2 inflection

= 0 → TP is a point

Step 6: Determine the y co- ord of the optimum solution found in step 5: f(x)|x = x co − ord Step 7: present answer as: The function equation is max/min when x = x co − ord. The max/min value is y = y co − ord.

**INDEFINITE INTEGRAL**

**COMMON INTEGRALS**

• ∫x dx

Equation Integral The answer will be an equation and remember

∫xndx to put a ‘+c’ at the end.

**DEFINITE INTEGRAL**

• ∫ x dx

n xn+1 + 1

+ c [n ≠ −1]

∫f'(x)×[f(x)]n dx

[f(x)]n+1 n + 1

+ c [n ≠ −1]

b a Where:

∫ ef(x) dx

• a – lower bound

• b – upper bound The answer will be a single number (all other variables are eliminated).

f′(x) ef(x)

+ c

∫

f'(x) f(x)

dx ln(f(x)) + c

∫sin(x) dx −cos(x) + c

∫cos (x) dx sin(x) + c

**RECTILINEAR RULES**

• Change in displacement between times a and b: ∫ a b

v dt

• Distance travelled between times a and b: ∫ a b

|v| dt

• Object changes direction whenever v = 0

• Object returns to the starting position whenever s = 0

**FUNDAMENTAL THEOREM OF CALCULUS**

•

dx d

(∫ a x

f(t)dt

) = f(x)

• ∫ a b

f′(x)dx = f(b) − f(a)

**A rectangular prism has total surface area of 6480cm2. Its width is x cm, length is 2.5x cm and height is h cm. Determine x that maximises the volume. V = lwh V = (x)(2.5x)(h) SA = 2lw + 2wh + 2hl = 6480 6480 = 5x2 + 7xh**

h =

**A rectangular box is to be made from a sheet of metal with squares of length x to be cut from the corners. If the sheet of metal is 65cm wide and 100cm long, determine the value of x that will maximise the volume of the box. V = lwh → There are 4 variables in this equation, we need to eliminate 2 variables by substitution: l = 100 − 2x, w = 65 − 2x and h = x V = (100 − 2x)(65 − 2x)x = 4x3 − 330x2 + 6500x dV dx**

x = −20.7846,20.7846 Lengths can’t be 0. Hence x = 20.7846 gives max volume.

Substituting x = 20.7846 into V, max volume is 32067.68m3

d dx

= 12x2 − 660x + 6500

Solving for when

x x

x x

6480 − 5x2 7x

∴ V = (x)(2.5x)(

dV dX

= 0: x = 12.85,42.15

d

x

x

x

x

100

6480 − 5x2 7x

2

dx

V

@x = 12.85 = −351.6 ∴ maximum

d

)

dV dx

2

2

dx Sub V 2

@x x = 42.15 = 351.6 ∴ minimum

= 12.85 to find max V = 37522cm3 ∴ The volume is maximised when x = 12.85cm. The maximum volume is 37522cm3.

−75x2 =

14

+

16200 7 Solving for when

dV dx

= 0

**AREA UNDER A CURVE USING RECTANGLES**

• Area under a curve is determined by the definite integral but can be estimated using rectangles.

• Estimate of area between x = a and x = b is: (Overestimate + Underestimate)/2

Estimate area between x = 2 and x = 9 of y = 0.2x2

2 3 4

5 6 7 8 9 Overestimate: 0.2(2)2 + ⋯ + 0.2(9)2 = 56.8 Underestimate: 0.2(1)2 + ⋯ + 0.2(8)2 = 40.8 Estimate Area: 56.8 + 2

40.8

= 48.8

**(a) Part of the curve f(x) = x2 − 3 is shown below. A value of k exists such that the area of the region marked A is equal to the area of the region marked B. Determine value of k as an exact number.**

Area A = |∫ 0 3

x2 − 9 dx

| = 18

Area B = ∫ 3 k

x2 − 9

dx

∴ ∫ 3 k

x2 − 9

dx = 18

g(x) + f(x) = 0 g(x) = −f(x) f(x) − −f(x) = 2f(x) ∫ |f(x) − g(x)|dx Solving for k gives k = 3√3

**(b) Define ∫ |f(x)|dx**

k 0

k 0

∫ 0 k

|2f(x)|dx

= 2∫ 0 k

|f(x)|dx From Part A = 2(2A) = 4A

**in terms of A.**

∫ 0 k

|f(x)|dx

= A + B

But A = B so ∫ 0 k

|f(x)|dx

= 2A

B

**(c) g(x) is another function**

A

k

**such that g(x) + f(x) = 0. Use this to show that:**

∫ 0 k

|f(x) − g(x)|dx = 4A

**.**

a v

2 3x (∫ 0

1 2 + − t t

dt

)

d dx

x

3 (∫ √t2 + 1 dt

) sin(x)

sub sin(x) into t:

= √(sin (x))2 + 1

**Determine f(x) with the following conditions:**

• F(x) = ∫ f(t)dt

F(x) formula to solve for c: F(3) = ∫ f(t)dt

Sub 3x2 into t:

Sub x3 into t:

Multiply by the

=

= √(x3)2 + 1

derivative of sin (x):

= √x6 + 1

= cos (x)√(sin (x))2 + 1

Multiply by the

Subtract the 2nd answer

derivative of x3:

from the 1st answer:

= 3x2√x6 + 1

= 3x2√x6 + 1

Repeat steps 1 and 2:

− cos (x)√(sin (x))2 + 1

x

0

3

= 5

0

5 = ∫

•

3

t

2

• F(3) d dx2 2

F

= = x 5.

+ 5

Using 0

ClassPad 2

to solve: c = −7.33 ∴ f(x) =

+ 5t + c dt

1 + 3x2 2 − 3x2 Multiply by the derivative of 3x2:

dF dx

= 6x (

t

2 2

+ 5t − 7.33

*Tip: If you are getting stuck on these harder questions, break down all of the derivatives given in terms of f(x) and dy/dx.*

QUESTIONS WITH FUNCTIONS AS LIMITS Step 1: sub the limits into t. Step 2: multiply the answer by the derivative of the limit. (Note: for questions with 2 limits, do steps 1 and 2 twice).

= f(x)

Hence Integrating d dx2 2

F

= f′(x) f′(x) to = get x + f(x):

5

f(x) = ∫x + 5dx =

1 2 + − 3x2 3x2

)

=

2 x

2

+ 5x + c

As

6x + 18x3 2 − 3x2

dF dx

= f(x), we can use the

*Integrate*

*Differentiate*

**G**

s

**9 f(x) is shown: A C**

(a) ∫ −10 9

**Determine ∫ −10**

f(x)dx f(x)dx

= A − B + c = 6

**(d) Determine**

∫ −10 −5

f(x) − 2dx (b) Determine ∫ 3f(x)dx

= ∫ f(x)dx B

**Roots are −10, −5 and 9.**

9 0 ∫ 3f(x)dx = 3×C = 9

−5

• A = 4

9 0

−10

− ∫ −5

2dx

• B = 1

**(c) Determine ∫ f(x)dx**

• C = 3

−5 9 = −∫ f(x)dx = −C + B = −2

−10 = A − [2x]

−5 −10 = 4 − 10 = −6

Determine the volume between the curves f(x) = ln (x) and g(x) = (x − 4)2 Step 1: Determine the points of intersection between the two curves by solving. f(x) = g(x) → ln(x) = (x − 4)2 x = 2.96,5.29

9 −5

Step 2: pick a number between the two solved x values (e.g. 4) and substitute it into both equations to determine the upper function. f(4) = 1.39 and g(4) = 0 Hence f(x) is the upper curve and g(x) is the lower curve.

Step 3: determine the integral that calculates the area between two curves ∫ a

b

upper curve −

b a

using: ∫ lower curve b = ∫ f(x)dx −

*Note: the limits are the same for both*

a

*integrals. b ∫ g(x)dx*

5.29

a

= ∫ ln(x)dx −

2.96

5.29 ∫ (x − 4)2dx

2.96

= 2.18

**(b) Find the formula for acceleration.**

a =

dt d

(v)

a a =

= dt

−6t d

(−3t2 − 2

− 2t + 5)

**(a) Acceleration of a body is a = 49 − 8t where motion is measures in m/s. After 5 seconds the particle is instantaneously stationary. Find the formula for velocity.**

v = ∫a dt

**(b) Find the distance v = ∫49 − 8t dt**

**travelled in the first 10 v = 49t − 4t2 + c**

seconds. 0 = 49(5) − 4(5)2 + c Solve for c: c = −145

= ∫ |59t − 4t2 − 145| dt

∴ v = 59t − 4t2 − 145

**A body has initial displacement of 10m and velocity v = t2 + 3t. Find the displacement when it has velocity of 63m/s?**

Solve 63 = t2 + 3t → t = −9.58,6.58

s = ∫v dt = ∫t2 + 3t dt =

t

3

3

+

3t 2

2

+ c

10

10 =

(0) 3

3

+

3(0)

2

2

+ c → c = 10

0 = 578.13 metres

∴ s =

t

3 3

+

3t 2

2

+ 10 and s(6.58) = 169.9m

DOUBLING TIME Time taken for amount to increase by 100% (i.e. A = 2A

0

x d dx

(∫ t2dt

) 0 = x2

d dx

x

0

)

= ln (x)

d dx

(∫ ln(t)dt

x (∫ e2tdt

) 0 = e2x

d dx

x (∫ √tdt

) 0 = √x

).

**(a) The temperature of the south pole over t years has the equation T = A + Be−kt. The initial temp is −50°C and the long term temp is −20°C, find A and B. −50 = A + Be−k(0) → −50 = A + B −30 = A + Be−k(∞) A = −30 and B = −20 (b) Determine k if after 10 years the temperature is −43.5°C −43.5 = −30 − 20e−k(10) → k = 0.0393**

**Net of Box (corners will be cut and folded)**

**INTEGRAL RULES**

• Swapping limits:

b

a ∫ f(x) =

− ∫ f(x)

• Constant a

in an b

Integral:

∫ axn dx = a∫ xn dx

• Area under a curve that goes below the x-axis:

b ∫ |x| dx

• Area a

between 2 curves:

∫ a

b

upper curve

−

∫ a

b

lower curve

**(a) Displacement of a body is given by s = −t3 + at2 + bt + 3 where t is the time in seconds. The body is temporarily at rest when t = 1. The initial velocity is 5 m/s. What is a and b?**

**(a) The population of a small island is P = 40 − 10e−kt, show that dP/dt = k(P − 40).**

dP

Hence, dt

= 10ke−kt dP

& P = 40 − 10e−kt

(b) dP dt

= If dt

= (−P + 40)×k

k(P − 40)

**k = 0.02, find the rate at which the pop is changing when the pop is 30. dP dt**

= 0.02(30− 40) = −0.2

**U**

**N**

**D**

**E**

**R**

**E**

**S**

**T**

**I**

**M**

**A**

**T**

**I**

**N**

**G**

**A**

**N**

**D**

**O**

**V**

**E**

**R**

**E**

**S**

**T**

**I**

**M**

**A**

**T**

**I**

**N**

(−t3 v =

+ at2 + bt + 3) v = −3t2 + 2at + b When t = 0,v = 5 ∴ b = 5 & v = −3t2 + 2at + 5 0 = −3(1)2 + 2a(1) + 5 ∴ a = −1

6 5

d dt

**PROBABILITY AND RANDOM VARIABLES**

**P**

**ATAR Math Methods Units 3 & 4 Page 3 / 4 Created by Anthony Bochrinis - Sharpened®**

**R**

**O**

**B**

**A**

**B**

**I**

**L**

**I**

**T**

**I**

**S**

**C**

**R**

**E**

**T**

**E**

**R**

**A**

**N**

**D**

**O**

**E**

**R**

**N**

**O**

**U**

**L**

**L**

**S**

**C**

**O**

**N**

**T**

**I**

**N**

**U**

**O**

**U**

**S**

**R**

**A**

**N**

**D**

**O**

**C**

**L**

**A**

**S**

**S**

**P**

**A**

**S**

**U**

**N**

**I**

**F**

**O**

**R**

**E**

**N**

**O**

**R**

**M**

**A**

**E**

**R**

**N**

**O**

**U**

**L**

**L**

**I**

**&**

**B**

**I**

**N**

**O**

**M**

**I**

**A**

**Y**

**R**

**U**

**L**

**E**

**S**

**&**

**L**

**A**

**W**

**S**

**M**

**V**

**A**

**R**

**I**

**A**

**B**

**L**

**E**

**S**

**(**

**D**

**R**

**V**

**I**

**D**

**I**

**S**

**T**

**R**

**I**

**B**

**U**

**T**

**I**

**O**

**N**

**M**

**V**

**A**

**R**

**I**

**A**

**B**

**L**

**E**

**S**

**(**

**C**

**R**

**V**

**)**

**D**

**B**

**E**

**R**

**N**

**O**

**U**

**L**

**L**

**M**

**D**

**I**

**S**

**T**

**R**

**I**

**B**

**U**

**T**

**I**

**O**

**N**

**L**

**D**

**I**

**S**

**T**

**R**

**I**

**B**

**U**

**T**

**I**

**O**

**N**

**L**

**Q**

**U**

**E**

**S**

**T**

**I**

**O**

**N**

**)**

**ABOUT DRV**

• Discrete distributions cover events that can be counted.

• It is only measured in integers (whole numbers).

• For example, counting how many students there are in each class of a school.

**CLASSPAD BERNOULLI To find:**

• p(x)

• P(a ≤ x ≤ b)

• the value of k given P(X ≤ k) Use the Binomial Distribution commands to the right and set all instances of n to 1.

**ABOUT UNIFORM**

• A Uniform distribution has constant probability.

• E.g. a volcano erupts every hour. You arrive there at random and wait 30 minutes, what is the chance it erupts?

**PROBABILITY NOTATION**

• ∪ → Union (or)

• ∩ → Intersection (and)

• | → given

• Á or A → complement

• ∅ → null set

• ∈ → element of

• ⊂ → subset

**ABOUT BERNOULLI**

• A Bernoulli trial is a binomial distribution with 1 trial.

• A Bernoulli trial has only two possible outcomes, which we may term “success” or “failure.”

• E.g. tossing a coin is a Bernoulli trial: you can either get heads or tails.

**ABOUT NORMAL**

• Normal distribution has the iconic “Bell Curve” shape which means that data closer to the mean has a higher chance of occurring.

• E.g. finding faults in cars from an entire factory.

**(a) Determine the values of a and b in the following discrete distribution if E(X) = 0.20**

x 0 1 2 3 4 p(x) 0.85 0.12 a b 0.005 Equation 1: 0.12 + 2a + 3b + 0.2 = 0.2 Equation 2: 0.85 + 0.12 + a + b + 0.005 = 1 ClassPad simultaneous solve: a = 0.015 and b = 0.01

**Y is a uniform distribution with a = 1 and b = 5. Determine the value of k in the following equations: (a) P(X > k|X < 3) = 0.5 P(k<X<3) P(X<3)**

ANALYSING GRAPHS A graph is suited for L

Binomial if it is either negatively skewed (long left tail) or positively skewed (long right tail). As n increases, graphs become more symmetrical (normal distribution).

**The probability of a successful trial is 0.4, how many trials are needed to ensure that the probability of 3 or more successes is exceeds 0.75? X~B(n,0.4) and P(X ≥ 3) > 0.75**

Method 1: Using trial and error for different values of n on ClassPad: binomialCDF(3,∞,n, 0.4) → when n = 9,CDF = 0.7682 ∴ 9

Method 2: P(X = Solving ( n 0 ≥ )(0.6)n 4) for > n 0.75 + on ( n 1 ClassPad, = )(0.4)(0.6)n−1 P(X = 0) n + ≈ P(X 9 + (

n = 2

) 1) (0.4)2(0.6)n−2 + P(X = 2) > = 0.75 0.75

**ABOUT CRV**

**Z is a CRV and is graphed**

• CRV’s cover events that

**on the set of axes below: can be measured.**

• Measures with decimal values (exact numbers).

• E.g measuring height of people in a city in cm.

**Determine f(z). CRV TYPES**

• Uniform Distribution

f(z) = {

• Normal Distribution

8

7

1 b − a

a b

**S**

**(a) Find the probability that a student passes a multi-choice test with 10 questions and 4 options per question by guessing answers? X~B(10,0.25)**

P(X ≥ 5) = P(5 ≤ X ≤ 10) = 0.0781

**(b) If the class has 15 students, what is the probability that at least 4 pass by guessing? X~B(15,0.0781) P(X ≥ 4) = P(4 ≤ X ≤ 15) = 0.0252**

1 5

2x − 2 1 ≤ x ≤ 5 −4x + 28 5 ≤ x ≤ 7 0 elsewhere

FIND P(C ≤ X ≤ D)

∫

**X is a uniform distribution**

C

D

b − 1

a

dx

**with a = 10 and b = 20. Determine the following:**

FIND P(X ≥ C|X ≤ D)

∫

(a) P(15)

D

1 C

b − a ∫

dx

D

1 A

b − a

**FIND K GIVEN**

= 0 (note: singular probabilities

= 0.5 of a CRV is always equal to 0). (b) P(X ≥ 14) X~U(10,20) = ∫

P(k < X < 3) = 0.25 ∴ k = 2 (b) P(X > 2|X < k) = 0.5 P(2<X<k) P(X<k) P(X ≤ K)

∫

dx

K

a

20

1 14

20−10

dx = 0.6 (c) P(X ≥ 14|X ≤ 18)

= 0.5

P(2 < X < k) = 0.5P(X < k) b − 1

a

dx = P(X ≤ K)

= ∫

14

18

20−10 1

18

1 10

20−10 dx/ ∫

dx =

0.5

Using trial and error for values of k: k = 3

**NORMAL NOTATION X~N(μ, σ2) where:**

• μ – mean

• σ – S.D.

**NORMAL RULES**

• E(X) = μ

• Var(X) = σ2

• σ = σ

**CRV RULES**

• ∫p(x)dx = 1

• f(x) ≥ 0

• P(X > a) = P(X ≥ a)

• P(X = a) = 0

• P(a ≤ X ≤ b) = ∫ b

p(x)dx

• • E(X) Var(X) = ∫ = −∞ ∞

∫ −∞ ∞ xp(x)dx (x − a

μ)2p(x)dx where μ = E(X)

**UNIFORM NOTATION X~U(a,b) where:**

• a – lower bound

• b – upper bound

**UNIFORM RULES**

• f(x) =

b−a 1

• E(X) =

1 2

(a + b)

• Var(X) =

12 1

(b − a)2

• σ = √

12 1

(b − a)2

• p(x) = 0

**PROBABILITY RULES**

• 0 ≤ P(A) ≤ 1

• P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

• P(A|B) =

P(A∩B) P(B)

or P(B|A) =

P(A∩B) P(A)

• P(A) = P(Á) = 1 − P(A)

• For independent events: P(A ∩ B) = P(A)×P(B)

**E**

**BERNOULLI NOTATION X~B(p) where:**

• p – probability of success

**BERNOULLI RULES**

• p(x) = {

1 − p p for x = 1 for x = 0

• E(X) = p

• Var(X) = p(1 − p)

• σ = √p(1 − p)

If E(X) = 5 and Var(X) = 2 (a) Determine E(X + 11) = E(X) + 11 = 5 + 11 = 16

(b) Determine E(1 − 2x) = 1 − 2E(X) = 1 − (2×5) = −9 (c) Determine Var(3X + 1) = 32Var(X) = 9×2 = 18

**DRV TYPES**

• Bernoulli Distribution

• Binomial Distribution DRV RULES

• ∑p(x) = 1

• 0 < p(x) < 1

• E(X) = ∑ i

p

i

x i • Var(X) = ∑ p

(x

− μ)2 D

i i

i

**I**

**C**

**L**

**A**

**S**

**S**

**P**

**A**

**(a) The chance of an apple being rotten in a delivery is 0.1. Find the probability that of 6 apples, 1 is rotten. = 0.95×0.1 = 0.0590**

**(b) Find the probability that exactly one of six apples chosen from the box are found to be rotten. X~B(6,0.1) = P(X = 1) = 0.3543**

**D**

**B**

**I**

**N**

**O**

**M**

**I**

**A**

FIND P(X) binomialPDF(x,n,p) Where:

• binomialPDF is found in Main App → Action → Distribution → Discrete

• x is the number of successful trials.

**S**

**N**

**O**

**R**

**M**

**A**

**L**

**C**

**L**

**A**

**S**

**S**

**P**

**A**

**D**

**U**

**N**

**I**

**F**

**O**

**R**

FIND P(A ≤ X ≤ B) normCDF(A, B,σ,μ) Where:

• normCDF is found in Main App → Action → Distribution → Continuous

FIND K GIVEN P(X ≤ K) invNormCDF("TS",P(X ≤ K),σ,μ) Where:

• invNormCDF is found in Main App → Interactive → Distribution → Inverse

**M**

**C**

**L**

**A**

**S**

**S**

**P**

**A**

**C**

**R**

**B**

**I**

**N**

**O**

**M**

**I**

**A**

**V**

**Q**

**U**

**E**

**S**

**T**

**I**

**O**

**N**

**L**

**D**

**I**

**S**

**T**

**R**

**I**

**B**

**U**

**T**

**I**

**O**

**N**

**D**

**R**

(a) X is a binomial variable. Determine the value of parameters n and p if E(X) = 21 and Var(X) = 6. 3. E(X) = 21 = np & Var(x) = 6.3 = np(1 − p) ClassPad simultaneous solve: n = 30 and p = 0.7 (b) Determine P(X ≥ 10|X ≤ 15) X~B(30,0.7)

=

P(X ≥ P(X 10 ≤ ∩ 15)

X ≤ 15)

P(10 ≤ X ≤ 15) P(X ≤ 15) = 0.9996

**X is a CRV. It is known that P(X > 5) = 0.6 and X has a probability density function of:**

f(x) = {

=

**Y is a CRV and has a probability density function of:**

**ax + b 0 ≤ x ≤ 10 0 elsewhere Determine the values of a and b.**

f(y) = {

2y2 0 + 3 0 ≤ x ≤ 2 elsewhere Determine E(Y) and Var(Y).

Equation 1: ∫ 0

10

ax + bdx = 1

Equation 2: ∫ 5

10

ax + bdx = 0.6 ClassPad simultaneous solve:

E(Y) = ∫ 0 2

(y)(2y2 + 3)dy E(Y) = 14 Var(Y) = ∫ (y − 14)2(2y2 + 3)dy

a = 0.008 and b = 0.06

2 0 Var(Y) = 1850.1333

FIND P(A ≤ X ≤ B) binomialCDF(A,B,n,p) Where:

• binomialCDF is found in Main App → Action → Distribution → Continuous

• A is the lower bound.

• B is the upper bound.

**V**

**Q**

**U**

**E**

**S**

**T**

**I**

**O**

**N**

**S**

**ABOUT BINOMIAL**

• A Binomial distribution is when you perform more than 1 independent Bernoulli trial.

• The Binomial distribution counts the number of success in an experiment with trials.

• For example, tossing a coin repeat times and counting the number of heads flipped.

**X**

**P**

**E**

**C**

**T**

**E**

**D**

**V**

**A**

**L**

**U**

(a) Is the following distribution discrete? x -1 0 1 2 p(x) 0.3 0.2 0.1 0.4 Yes, as all probabilities add to 1. (b) Is the following distribution discrete? x 0 1 2 3 p(x) -0.1 0 0.5 0.6 No, as p(x) cannot be negative.

**B**

**E**

**A**

**N**

**D**

**V**

**A**

**R**

**I**

**A**

**N**

**C**

**E**

**EXPECTED VALUE AND VARIANCE**

• E(X) = μ = Expected Value

• Var(X) = Variance

• √Var(X) = S.D.

**Comparing E(X) and Var(X)**

• Var(X) = E(X)2 − [E(X)]2

**U**

**N**

**I**

**F**

**O**

**R**

FIND K GIVEN P(X ≤ K) invBinomialCDF(P(X ≤ k),n, p) Where:

• invBinomialCDF is found in Main App → Action → Distribution → Inverse

• n is the number of trials.

• p is the probability of success.

**M**

**Q**

**U**

**E**

**S**

**T**

**I**

**O**

**N**

**D**

**Z**

**-**

**S**

**C**

**O**

**R**

**E**

**A**

**N**

**BINOMIAL NOTATION X~B(n,p) where:**

• n – number of trials

• p – probability of success BINOMIAL • p(x) = ( n x

RULES )(p)x(1 − p)n−x

• E(X) = np

• Var(X) = np(1 − p)

• σ = √np(1 − p)

**D**

**6**

**8**

**/**

**9**

**5**

**/**

**9**

**9**

**.**

**7**

**R**

**U**

**L**

EFFECTS OF LINEAR CHANGE If X is a random variable and Y = aX + b then:

• E(X) = aE(X) + b

• Var(X) = a2Var(X) Where a and b are constants.

**Z-SCORE**

• Z~N(0,1) Where: Z =

X−μ σ Z-Scores simplifies all normal distributions to a mean of 0 and a S.D. of 1. Z-scores indicate how many S.D.’s away from the mean each score is.

**B**

**B**

**I**

**N**

**O**

**M**

**I**

**A**

**68/95/99.7 RULE**

**L**

**G**

**R**

**A**

**P**

**H**

**B**

**E**

**R**

**N**

**O**

**U**

**L**

**L**

**I**

**V**

**S**

**68%**

−σ

**σ 95%**

−2σ 2σ

**99.7%**

−3σ

3σ

**.**

**B**

**I**

**N**

**O**

**M**

**I**

**A**

**L**

**BERNOULLI VS. BINOMIAL**

• If the number of trials is equal to 1, the distribution is Bernoulli.

• If the number of trials is more than 1, the distribution is Binomial.

**B**

**PROBABILITY AND RANDOM VARIABLES**

**INTERVAL ESTIMATES**

**YOUR NOTES AND EXAMPLES**

**ATAR Math Methods Units 3 & 4 Page 4 / 4 Created by Anthony Bochrinis - Sharpened®**

**E**

**N**

**T**

**R**

**A**

**L**

**L**

**I**

**M**

**I**

**O**

**N**

**F**

**I**

**D**

**E**

**N**

**C**

**E**

**I**

**N**

**T**

**E**

**R**

**V**

**A**

**L**

**N**

**O**

**R**

**M**

**A**

**T**

**T**

**H**

**E**

**O**

**R**

**E**

**M**

**S**

**A**

**N**

**D**

**M**

**A**

**R**

**G**

**I**

**N**

**O**

**F**

**E**

**R**

**R**

**O**

**R**

**L**

**Q**

**U**

**E**

**S**

**T**

**I**

**O**

**N**

**S**

**CONFIDENCE INTERVAL RULES**

• (p̂ − Z√

p̂(1−p̂) n

, p̂ + Z√

p̂(1−p̂) n

)

• p̂ ± E Where:

• Z → Z-Score for a given confidence interval (refer to table below for common z-scores):

**% Confidence Interval**

**Z-Score**

99% CI 2.58 95% CI 1.96 90% CI 1.645

Custom confidence interval: z

c

= −1× invNormCDf("C", c, 1, 0) Where: c → CI% as a decimal

If X~N(μ,σ2) such that the mean is twice the variance and P(X > 10) = 0.3. Find μ and σ. μ = 2σ2,∴ X~N(2σ2,σ2). Note: use Z-Scores. invNormCDf("L",0.3,1,0 ) = 0.5244

Z =

X − σ

μ

, 0.5244 =

10 σ

− μ

,0.5244 =

10 − σ

2σ2

solve(0.5244 =

10−2σ

σ

2

) σ = 2.11 or − 2.37 and μ = 2(2.11)2 = 8.89

6

**C**

**CENTRAL LIMIT THEOREM RULES**

• Regardless of the original distribution, if the number of independent random samples of the experiment is a large number (n ≥ 25), the data can be modelled using a normal distribution.

**ORIGINAL DISTRIBUTION IS UNKNOWN**

• As n ≥ 25, the distribution becomes normal with the following parameters:

o Mean stays the same: X̅ o Standard Deviation changes to:

√n σ

**ORIGINAL DISTRIBUTION IS BERNOULLI**

• As n ≥ 25, the distribution becomes normal with the following parameters:

o Mean stays the same: p

o Standard Deviation changes to:√

**p(1−p) n SAMPLE NOTATION**

• μ and p are statistics of the original population.

• X̅ and p are statistics of the sample population.

**MARGIN OF ERROR RULES**

• E = Z√

p̂(1−p̂) n

• E ∝

√n 1

CHANGING CONFIDENCE INTERVALS If you are changing the confidence interval of a question (i.e. you are given a 95% CI and you need to determine a 99% CI), follow these steps:

Step 1: determine p: p =

lower bound + 2

upper bound

Step 2: determine E: E = upper bound − p Step 3: determine E

new

: E

new

=

z z

new

old

×E

Step 4: determine new confidence interval: New CI = p ± E

new

**QUANTILE (a) The time in hours that a brand of light globe operates before going out**

**RULES is normally distributed with a mean of 9000 hours and a standard**

• P(X < t

a

**) = a where: 0 < a < 1 deviation of 450 hours, what is the probability of a light globe lasting more**

**• The ath percentile is the score that than 8000 hours given that it does not last more than 10000 hours?**

a% of the population lies below. X~N(9000,4502) → P(X ≥ 8000|X ≤ 10000) =

P(8000≤X≤10000) P(X≤10000)

= 0.9867

**(b) Determine how many life hours are exceeded by 55.6% of all light**

**Percentile Probability**

25th P(X < t

a

**) = 0.25 globes.**

50th P(X < t

a

) = 0.50 P(X < k) = 0.556 → invNormCDF("L",0.556,450,9000) = 9063.3759 hours

75th P(X < t

a

) = 0.75

**Z-SCORE FOR AN UNKNOWN DISTRIBUTION**

Z =

X̅ − σ

μ

√n

**Z-SCORE FOR A BERNOULLI DISTRIBUTION**

Z =

√

p̂(1 p̂ − n − p

p̂)

**NATURE AS N CONTINUES TO INCREASE**

• As n → ∞, the distribution approaches the standard normal distribution.

**(a) In one particular store, 18% of pizzas are overcooked. In a sample of 150 pizzas, describe the distribution and state the mean and standard deviation.**

p = 0.18 and s = √

**(a) 23% of Australians are left handed. If a sample of 40 Australians are surveyed, what proportion of these samples are expected to contain less than 20% of left-handers?**

0.18(1−0.18) 150

= 0.0314

Hence, p~N(0.18,0.03142)

p~N(p, s2) where p = 0.23 and s = √

0.23(1−0.23) 40 p~N(0.23,0.06652) and P(p < 0.2) = 0.3257 (b) Determine the probability that a

**(b) What proportion of these samples are point estimate for the proportion of**

**expected to contain between 10% and 15% of overcooked pizzas exceeds 0.21.**

left-handers? P(p > 0.21) = 0.1697

P(0.10 < p < 0.15) = 0.0892

• Calculate the width of a confidence interval: Width = 2E

**What is the margin of error on a 99% confidence interval of (0.25,0. 32)?**

E =

*Note: p does not have to 0.32 − 2*

0.25

= 0.035

*be given to determine the margin of error.*

**C**

**O**

**N**

**T**

**U**

**N**

**U**

**I**

**T**

**C**

**E**

**N**

**T**

**R**

**A**

**L**

**L**

**I**

**M**

**I**

**Y**

**C**

**O**

**R**

**R**

**E**

**C**

**T**

**I**

**O**

**N**

**T**

**T**

**H**

**E**

**O**

**R**

**E**

**M**

**Q**

**U**

**E**

**S**

**T**

**I**

**O**

**N**

**S**

**C**

**O**

**N**

**F**

**I**

**D**

**E**

**N**

**C**

**E**

**I**

**N**

**T**

**E**

**R**

**V**

**A**

**L**

**CONTINUITY CORRECTION RULES**

• When using a discrete distribution and n ≥ 25, it can be modelled using a normal distribution (continuous).

• When changing from a discrete distribution to a continuous distribution, the probabilities you calculate change slightly according to the table:

**S**

**A**

**N**

**D**

**M**

**A**

**R**

**G**

**I**

**N**

**O**

**F**

**E**

**R**

**R**

**O**

**R**

**Q**

**U**

**E**

**S**

**T**

**I**

**O**

**N**

**S**

**A 90% Confidence Interval is (0.38,0.45). Determine a 95% Confidence interval.**

p =

0.38 + 2

0.45

= 0.415

E = 0.45 − 0.415 = 0.035

E

new

=

1.645 1.96

×E = 0.0417

95% CI = 0.415 ± 0.0417 = (0.3733,0.4567)

**How many times larger is the margin of error of a sample of 1225 compared to a sample of 11025? E ∝**

√1225 1

1 35

E ∝

=

∴ 3 times

1 √11025

=

105 1

as large.

**(a) In a random sample of 400 people, 129 we male. Calculate a 90% confidence interval. p =**

129 400

= 0.3225

90% CI = 0.3225 ± 1.645√

**0.3225(1−0.3225) 400 90% CI = (0.2898,0.3552) (b) What is the largest size sample of people that would have to be taken in order for a width of a 99% confidence interval to be 0.1 or less?**

E = z√

p(1 n

− p)

→

0.1 2

= √

0.3225(1 n

− 0.3225)

Solving for n: n = 87.3975 ≈ 88 people are needed.

**Q**

**U**

**A**

**N**

**T**

**I**

**L**

**E**

**S**

Discrete Continuous P(X = k) P(k − 0.5 < X < k + 0.5) P(X > k) P(X > k + 0.5) P(X ≥ k) P(X > k − 0.5) P(X < k) P(X < k − 0.5) P(X ≤ k) P(X < k + 0.5)

**C**